Models of Set Theory II - Winter 2017/2018

Prof. Dr. Peter Koepke, Ana Njegomir Problem sheet 11

Problem 1 (5 points)**.** Suppose that for every uncountable cardinal κ every stationary subset of $[\kappa]^\omega$ remains stationary in every $\mathbb{P}$-generic extension of M. Let G be an M-generic filter for $\mathbb{P}$.

(a) Show that for every countable set $X \in M[G]$ of ordinals there is a set $Y \in M$ such that Y is countable in M and $X \subseteq Y$.

(b) Conclude that $\mathbb{P}$ preserves $\aleph_1$.

Definition. A forcing notion $(\mathbb{P}, \leq)$ satisfies *Axiom A* if there is a collection of $(\leq_n)_{n\in\omega}$ of partial orderings of $\mathbb{P}$ such that $p \leq_0 q$ implies $p \leq q$ and for every $n \in \omega$, $p \leq_{n+1} q$ implies $p \leq_n q$, and

(1) if $\langle p_n \mid n \in \omega \rangle$ is a sequence such that $p_0 \geq_0 p_1 \geq_1 \dots \geq_{n-1} p_n \geq_n \dots$ then there is a q such that $q \leq_n p_n$ for all n;

(2) for every $p \in \mathbb{P}$, for every n and for every name $\dot{\alpha}$ for an ordinal there exist a $q \leq_n p$ and a countable set B such that $q \Vdash \dot{\alpha} \in \check{B}$

Problem 2 (5 points)**.** Show that every c.c.c. forcing satisfies Axiom A.

Problem 3 (5 points)**.** Prove that every σ-closed forcing satisfies Axiom A.

Problem 4 (5 points)**.** Show that $\mathfrak{b} \leq \mathrm{cof}(\mathfrak{d})$.

Please hand in your solutions on Monday, January 8 before the lecture.

Happy holidays!